

Relay Channel with Orthogonal Components and Structured Interference Known at the Source

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Abstract

A relay channel with orthogonal components that is affected by an interference signal that is non-causally available only at the source is studied. The interference signal has structure in that it is produced by another transmitter communicating with its own destination. Moreover, the interferer is not willing to adjust its communication strategy to minimize the interference. Knowledge of the interferer's signal may be acquired by the source, for instance, by exploiting HARQ retransmissions on the interferer's link. The source can then utilize the relay not only for communicating its own message, but also for cooperative interference mitigation at the destination by informing the relay about the interference signal. Proposed transmission strategies are based on partial decode-and-forward (PDF) relaying and leverage the interference structure. Achievable schemes are derived for discrete memoryless models, Gaussian and Ricean fading channels. Furthermore, optimal strategies are identified in some special cases. Finally, numerical results bring insight into the advantages of utilizing the interference structure at the source, relay or destination.

I. INTRODUCTION

Interference provides a major impairment for many current and envisioned wireless systems. Techniques that are able to mitigate interference are thus expected to be of increasing importance in the design of wireless networks. Two critical features of interfering signals can be leveraged to make the task of interference management more effective. The first is that interference is *structured*, as it typically arises from the transmissions of other wireless users. The second is that *information about the interference* can be obtained by wireless nodes in the vicinity of the interferer in a number of relevant scenarios. As an example, assume that the interferer employs retransmissions (HARQ) on its link. A node in the vicinity may be able to decode a prior retransmission and use this information in order to facilitate interference mitigation. Another scenario where interference information is conventionally assumed is cognitive radio.

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In this paper, we investigate interference mitigation strategies for a cooperative communication scenario in which a source communicates via an out of band relay to a destination in the presence of an external interferer. The interferer is not willing, or not allowed, to change its transmission strategy to reduce interference on the destination. The source is able to obtain information about the interferer signal prior to transmission in the current block. We are interested in studying effective ways to use such interference information at the source, in particular, the ones that leverage the structure of the interference.

The source can exploit the interference structure in a number of ways. For instance, the structure of the interference signal potentially allows the source to reduce the amount of spectral resources necessary for communicating interference information to the relay. A second way to take advantage of the interference structure is for the source, possibly with the help of the relay, to help reception of the interfering signal at the destination so that the destination can decode and remove the interference. In this work, we will explore these possibilities and assess the advantages of strategies that exploit the interferer's structure with respect to the techniques studied in [1] that assume an unstructured interferer.

A. Related Work

A simple model for the interference signal assumes that it is unstructured and, in particular, that it consists of an independent identically distributed (i.i.d.) sequence. This model is accurate, for instance, if the interference is the sum of the contributions of many interferers, all of comparable powers. In information-theoretic terms, an i.i.d. interference can be modelled as the “state” of a channel. The capacity of a state-dependent memoryless channel, where the state sequence (i.e., the interference) is available non-causally at the transmitter, is established by Gel'fand Pinsker in [2] (see also [3]). Costa [4] applied Gel'fand and Pinsker's (GP) result to the Additive White Gaussian Noise (AWGN) model with additive Gaussian state, giving rise to the so called Dirty Paper Coding (DPC) technique. DPC achieves the state-free capacity even though the state is not known at the receiver. It was shown in [5], [6], that this principle continues to hold even if the state is not Gaussian. However when there is no channel state information at the transmitter (CSIT), DPC can no longer achieve state-free capacity for AWGN with additive Gaussian state. This aspect for various assumptions on the channel gains was captured and studied in [7]-[11].

Extensions to the multiuser case were performed by Gel'fand and Pinsker in [12] and by Kim *et al.* in [13][14]. In particular, in [13][14] it is proved that for MACs multi-user versions of GP and DPC, referred to as multi-user GP (MU-GP) and multi user DPC (MU-DPC) respectively, achieve optimal performance. In [15], Somekh-Baruch *et al.* considered a memoryless two-user MAC, with the state available only to one of the encoders. The capacity region is shown to be obtained by generalized GP (GGP) and generalized DPC (GDPC). The scenario studied in this paper, but with an i.i.d. state is investigated in [1], [16] for a Discrete Memoryless (DM) and Gaussian relay channels with an in-band relay and [17] [18] for a DM and Gaussian relay channel with an out-of-band relay where lower and upper bounds on the capacity are derived.

With a single dominating interferer, interference structure can be utilized. This was recognized in [19], where a scenario in which a transmitter-receiver pair communicates in the presence of a single interferer is studied. It is shown therein that using GP coding, and hence treating the interference as if it were unstructured, it is generally suboptimal and *interference forwarding* with joint decoding at the destination can be beneficial [20]. This aspect is further studied in [21] for a MAC with structured interference available at one encoder, in [18] for a Gaussian relay channel with an out-of-band relay and in [22] for a cognitive Z-interference channel, where extensions of the techniques proposed in [19] are investigated.

B. Contributions and Organization

In this paper, we study interference mitigation techniques for the relay channel with orthogonal components [23] and with an external interferer whose signal is non-causally available only at the source. The relay channel with orthogonal components model is chosen due to its ability to model half-duplex communications and availability of capacity achieving strategies [23]. We propose several techniques for discrete memoryless, AWGN and Ricean fading channels that leverage interference structure to different degrees. We also establish optimality of specific transmission strategies for several special cases. Finally, numerical results bring insight into the advantages of interference mitigation techniques that exploit the interference structure.

II. SYSTEM MODEL

The scenario under study consists of a relay channel with an orthogonal source-to-relay link in the presence of an interferer. In this model, the source sends two different signals, one to the relay and

one to the destination with the help of the relay in orthogonal channels. The interference signal is available non-causally to the source as depicted in Fig. 1. We first consider a Discrete Memoryless Channel (DMC) version of the channel, which is described by the conditional probability mass functions (pmfs) $P_{Y_D|X_{SD}X_RX_I}$ and $P_{Y_R|X_{SR}X_R}$ where $Y_D \in \mathcal{Y}_D$, $Y_R \in \mathcal{Y}_R$, $(X_{SD}, X_{SR}) \in \mathcal{X}_{SD} \times \mathcal{X}_{SR}$, $X_R \in \mathcal{X}_R$ and $X_I \in \mathcal{X}_I$ are the destination (D) output, the relay (R) output, the source (S) input, the relay (R) input and the interference (I) signal, respectively. The pmf $P_{Y_D|X_{SD}X_RX_I}$ describes the stochastic relation between the signals transmitted by the source towards the destination (X_{SD}), by the relay (X_R), and by the interferer (X_I) and the signal received at the destination (Y_D). Similarly, the pmf $P_{Y_R|X_{SR}X_R}$ represents the relationship between the signals transmitted by the source towards the relay (X_{SR}) and by the relay (X_R) and the signal received at the relay (Y_R).

The source wishes to transmit a message W to the destination with the help of the relay in n channel uses. The message W is uniformly distributed over the set $\mathcal{W} = \{1, \dots, 2^{nR}\}$, where R is the rate in bits/channel use. The interferer employs a fixed (and given) codebook that is not subject to design. In particular, the codebook of the interferer is assumed to be chosen by the interfering terminal independently to communicate with some other destination which is not modeled explicitly. The message W_I of the interferer is assumed to be uniformly distributed over the set $\mathcal{W}_I = \{1, \dots, 2^{nR_I}\}$, where R_I is the interferer's rate in bits/channel use. We assume that the interferer's codebook is generated according to a pmf P_{X_I} . The generated codebook of the interferer is known to all nodes. Furthermore, the interferer's message W_I is known to the source. In the sequel we use the standard definitions of achievable rates and probability of error [24].

We also consider the AWGN scenario shown in Fig. 2. For this model, the input and output relations at time instant i are given as

$$Y_{R,i} = h_{SR,i}X_{SR,i} + Z_{R,i} \quad \text{and} \quad Y_{D,i} = h_{SD,i}X_{SD,i} + h_{RD,i}X_{R,i} + h_{I,i}X_{I,i} + Z_{D,i} \quad (1)$$

where the noises $Z_{D,i}$ and $Z_{R,i}$ are independent zero mean complex Gaussian random variables with unit variance, and $h_{SR,i}$, $h_{SD,i}$, $h_{RD,i}$ and $h_{I,i}$ are the complex valued channel gains accounting for propagation from the source to the relay ($h_{SR,i}$), from the source to the destination ($h_{SD,i}$), from the relay to the destination ($h_{RD,i}$), and from the interferer to the destination ($h_{I,i}$), respectively.

The codewords of the source X_{SR}^n and X_{SD}^n are subject to a total energy constraint nP_S and the codewords of the relay X_R^n is subject to power constraint nP_R as

$$\frac{1}{n} \sum_{i=1}^n (\mathbb{E} [|X_{SR,i}|^2] + \mathbb{E} [|X_{SD,i}|^2]) \leq P_S \quad \text{and} \quad \frac{1}{n} \sum_{i=1}^n \mathbb{E} [|X_{R,i}|^2] \leq P_R. \quad (2a)$$

We assume that the interferer codebook is generated i.i.d. with complex Gaussian distribution with zero mean and power P_I . We use the notation $\mathcal{C}(x) = \log_2(1+x)$.

For the AWGN model (1), we study the following two scenarios: (i) *No fading*: All channel gains remain constant over the entire coding block and are perfectly known to all nodes; (ii) *Ergodic fading*: All channel gains change in an ergodic fashion. The instantaneous values of channel gains are not known to the transmitters but are available at the receivers. Specifically, h_{SR} is known at the relay and h_{SD} , h_{RD} , h_I are known at the destination. Channel statistics instead are known at all nodes. In particular, we assume that channel gains h_{SR} , h_{SD} , h_{RD} and h_I are independent Ricean distributed with parameters K_{SR} , K_{SD} , K_{RD} , K_I , respectively.

III. ACHIEVABLE RATES FOR DM AND AWGN CHANNELS

While reference [1] focuses on achievable rates for the case where the interference signal X_I^n is i.i.d., here we concentrate on techniques that exploit the interference structure, as modeled in the previous section. The advantages of leveraging interference structure will be discussed in Sec. VI via numerical results through comparison with the techniques proposed in [1] (which will be also recalled below for completeness).

The proposed techniques are based on the following considerations. In [23], El Gamal and Zaidi prove the optimality of partial decode-and-forward (PDF) for the relay channel with orthogonal components in Fig. 1 without interference. Motivated by this, we assume that the relaying strategy for the source message is based on PDF. Specifically, the source message W is split into two independent messages, $W = (W', W'')$, where W' is sent through the relay and W'' is sent directly to the destination. The messages W' and W'' are uniformly distributed over the set $\mathcal{W}' = \{1, \dots, 2^{nR'}\}$ and $\mathcal{W}'' = \{1, \dots, 2^{nR''}\}$, respectively, and the total rate is $R = R' + R''$.

Interference mitigation is utilized either by the source only or by both the source and the relay in a cooperative fashion. In order to perform cooperative interference mitigation, the source needs to share the interference information with the relay. The structure of the interference plays an important role for the two phases of informing the relay of the interference and of interference mitigation towards the destination. We categorize the possible strategies in both phases as follows:

- *Communication of interference to the relay:* When the source chooses to inform the relay about the interfering signal, it has two options:
 - 1) *Digital interference sharing:* The structure of the interference is exploited as follows. The source encodes the interference index W_I into a codebook (not necessarily the same as the interferer's codebook) and sends it to the relay through the orthogonal source-relay ($S - R$) channel. The relay then decodes the interference index W_I .
 - 2) *Compressed interference sharing:* The structure of the interference is not used and the interference is treated as an i.i.d. sequence. Specifically, the source simply quantizes the interference sequence X_I^n and forwards the compressed description to the relay through the orthogonal source-relay channel. The relay hence recovers the interference sequence with some quantization distortion.
- *Interference mitigation at the destination:* There are several interference mitigation scenarios applicable to our model depending on the availability of interference information at the relay. We mainly concentrate on two approaches:
 - 1) *Structured approach:* The structure of the interference is exploited at the destination to decode and remove the interference signal. Decoding can be facilitated by having the source and/or the relay forward information about the interference to the destination. When the source does not inform the relay about the interfering signal, interference forwarding is performed by the source only. Otherwise, interference forwarding is done jointly by the source and the relay. In the AWGN channel, interference forwarding is performed by the source and/or relay by transmitting signals that are coherent with the interferer's signal, so that the correlation between transmitted signal and interference is positive;
 - 2) *Unstructured approach:* The structure of the interference is ignored at the destination and the interference is treated as an i.i.d. state. Interference precoding via GP, MU-GP or GGP for the DMC model, and DPC, MU-DPC and GDPC for the AWGN model, are utilized by the source only or by the source and the relay jointly depending on the availability of interference information at the relay. This class of techniques was extensively explored in [1] and will be considered here only in combination with the digital approach mentioned

above (not applicable in the unstructured model of [1]), and for reference.

Below, we list proposed achievable schemes based on the above categorization.

We only consider the scenario where the source and the relay cooperate for both source signal and interference mitigation. Strategies for which the source uses the relay only for signal forwarding, but not for interference management, are the special cases of the schemes below.

1) *Scheme (D,U) (Digital interference sharing, Unstructured approach)*: In this scheme, the source sends the interference digitally to the relay, so that the relay is fully informed about the interference sequence. In addition, the source also forwards part of the source message to the relay according to PDF. Then, the source and the relay follow the unstructured approach by jointly employing multi-user GP (MU-GP) [14] to forward the source message.

Proposition 3.1: For Scheme (D,U), the following rate is achievable for the DM model:

$$R_{(D,U)} = \max \min \begin{cases} I(U_S; Y_D | U_R) - I(U_S; X_I | U_R) + (I(X_{SR}; Y_R | X_R) - R_I)^+, \\ I(U_S U_R; Y_D) - I(U_S U_R; X_I) \end{cases} \quad (3)$$

where the maximization is taken over the input pmfs $P_{U_S U_R X_R X_{SR} X_{SD} | X_I}$ of the form $P_{X_{SR} | X_R X_I} P_{U_S U_R X_R X_{SD} | X_I}$, where U_S, U_R are finite-alphabet auxiliary random variables.

Sketch of the proof: The message W is split into two messages W' and W'' . The source conveys the message W' to the relay together with interference index W_I which leads to the constraint $R' \leq I(X_{SR}; Y_R | X_R) - R_I$. Since both the source and the relay have the interference knowledge, they are able to implement MU-GP [14] to send W' and W'' to the destination. Note that unlike [14], here the two encoders (source and relay) have the common message W' , so that the channel from the source and the relay to the destination is equivalent to the state (interference) dependent MAC with common message and informed encoders. An achievable rate region can be derived by following similar steps in [14][15], obtaining

$$R'' \leq I(U_S; Y_D | U_R) - I(U_S; X_I | U_R) \quad (4a)$$

$$R' + R'' \leq I(U_S U_R; Y_D) - I(U_S U_R; X_I) \quad (4b)$$

for some distribution $P_{U_S U_R X_R X_{SD} | X_I}$. Incorporating (4) with the constraint on R' gives us (3). \square

Proposition 3.2: For Scheme (D,U), the following rate is achievable for the AWGN model (1):

$$R_{(D,U)} = \max_{\substack{\rho_{W'}, \rho_{W''}, \gamma: \\ |\rho_{W'}|, |\rho_{W''}|, \gamma \in [0,1]}} \min \begin{cases} \mathcal{C}(P_{W''}) + (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - R_I)^+, \\ \mathcal{C}(P_{W''} + P_{W'}) \end{cases} \quad (5)$$

subject to $|\rho_{W'}|^2 + |\rho_{W''}|^2 \leq 1$

where $P_{W'} = (|h_{RD}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2$ and $P_{W''} = |h_{SD}|^2|\rho_{W''}|^2\gamma P_S$.

Sketch of the proof: The result is obtained from (3), where all inputs are chosen according to Gaussian distribution. Specifically, X_{SD} is allocated power γP_S , $0 \leq \gamma \leq 1$, and the remaining power $(1 - \gamma)P_S$ is allocated to X_{SR} . We set $X_{SD} = \rho_{W'}\sqrt{\gamma P_S}U_{W'} + \rho_{W''}\sqrt{\gamma P_S}U_{W''}$ and $X_R = \sqrt{P_R}U_{W'}$ where $U_{W'}$ and $U_{W''}$ are independent, zero mean, unit variance, complex Gaussian random variables and carrying the messages W' and W'' , respectively. Furthermore, $U_{W'}$ and $U_{W''}$ are independent of X_I . The source conveys W' to the relay at rate $R' \leq (\mathcal{C}(|h_{SR}|^2(1 - \gamma)P_S) - R_I)^+$ and the interference at rate R_I . MU-DPC is used by the source and the relay for transmission to the destination, where the precoding is done via U_S and U_R in (3) which are chosen to be linear combinations of (X_{SD}, X_I) and (X_R, X_I) as $U_S = X_{SD} + \alpha_S X_I$ and $U_R = X_R + \alpha_R X_I$ with inflation factors α_S and α_R and (X_{SD}, X_R) jointly complex Gaussian and independent of X_I . When the inflation factors are optimized the effect of the interference is completely eliminated at the destination similar to [14], leading to (5). We refer the readers to [4] and [14] for details on DPC and MU-DPC. \square

Remark 3.1: It is shown that the interference-free capacity region can be achieved by MU-DPC in [13] for Gaussian relay channel when the interference is non-causally available at both the source and the relay. Apart from the fact that we consider a relay channel with orthogonal components, the main difference with [13] is that the relay does not know the interference a priori but is informed about the interference through the orthogonal source-relay link. Note that the structure of the interference is essential in Proposition 3.2 in conveying the interference signal to the relay. However, this structure is not used in interference mitigation at the destination.

Remark 3.2: Once can also consider a scheme (D,S) in which the interference is digitally transmitted to the relay and the structured approach for decoding at the destination is used. Scheme (D,S) may lead to performance improvements over Scheme (D,U) for a DMC. However, for AWGN channels, Scheme (D,S) is inferior to Scheme (D,U), since Scheme (D,U) is able to completely remove the effect of interference at the destination via MU-DPC. We will observe in Sec. V and Sec. VI-B that, however, for fading channels MU-DPC typically fails to eliminate the effect of the interference at the destination completely and Scheme (D,S) may outperform Scheme (D,U).

2) *Scheme (C,U) (Compressed interference sharing, Unstructured approach):* With this scheme, studied in [1] and [17] for the general relay channel and relay channel with orthogonal components

respectively, the source sends the compressed interference signal and the part of the message to the relay and the unstructured approach is utilized for decoding at the destination. Achievable rate for Scheme (C,U) for our DM model can be obtained from [1, Corollary 1]. It can be extended to Gaussian case by using an approach similar to [1, Theorem 6] and taking the complex channel gains into account. The achievable rate for (C,U) for the AWGN model (1) can be written as

$$R_{(C,U)} = \max_{\substack{r_q, \rho_{W'}, \rho_{W''}, \rho_{W_I}, \gamma: \\ |\rho_{W'}|, |\rho_{W''}|, |\rho_{W_I}|, \gamma \in [0,1]}} \min \{ (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - r_q)^+, \mathcal{C}(P_{W'}) \} + \mathcal{C}(P_{W''}) \quad (6)$$

subject to $0 \leq r_q \leq \mathcal{C}(|h_{SR}|^2(1-\gamma)P_S)$ and $|\rho_{W'}|^2 + |\rho_{W''}|^2 + |\rho_{W_I}|^2 \leq 1$,

where $P_{W'} = (|h_{RD}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2 / (1 + \xi^2 D + P_{W''})$, $P_{W''} = |h_{SD}|^2 |\rho_{W''}|^2 \gamma P_S$, $D = P_I 2^{-r_q}$ and $\xi = |h_I| - |h_{SD}||\rho_{W_I}|\sqrt{\gamma P_S / P_I}$.

Remark 3.3: When $r_q = 0$ in (6), (C,U) boils down to the special case in which the relay is utilized only for source message cooperation and the source mitigates the interference by itself.

3) *Scheme (C,S) (Compressed interference sharing, Structured approach):* We propose two schemes in the class (C,S). For both schemes, the source informs the relay using compressed interference information, and the structured approach is used to mitigate interference at the destination. The schemes differ in the way the compressed interference information is used at the source, relay and destination nodes. In the first scheme, referred to as (C,S,1), the compressed interference information is used only to improve the reception of the interference signal at the destination by forwarding an “analog” version of the compressed interference. In the second scheme, referred to as (C,S,2), the compressed interference information is re-encoded by source and relay and decoded at the destination in a similar way as for standard compress-and-forward protocols for the relay channel (See, e.g., [25]).

Proposition 3.3: For Scheme (C,S,1), the following rate is achievable for the DM model:

$$R_{(C,S,1)} = \max \min \begin{cases} I(V; Y_D | U X_I) + (I(X_{SR}; Y_R | X_R) - I(X_I; \hat{X}_I))^+, \\ (I(V X_I; Y_D | U) - R_I)^+ + (I(X_{SR}; Y_R | X_R) - I(X_I; \hat{X}_I))^+, \\ I(V U; Y_D | X_I), \\ (I(V U X_I; Y_D) - R_I)^+ \end{cases}, \quad (7)$$

where the maximum is over all input pmfs $P_{UV\hat{X}_I X_R X_{SR} X_{SD} | X_I}$ of the form $P_{\hat{X}_I | X_I} P_{X_{SR} | X_R} P_U P_{X_R | U \hat{X}_I} P_{V | U X_I} P_{X_{SD} | V \hat{X}_I}$.

Sketch of the proof: The source quantizes the interference signal X_I^n into a reconstruction sequence \hat{X}_I^n at rate $I(X_I; \hat{X}_I)$ using some test channel $P_{\hat{X}_I|X_I}$ and sends the index of the quantized interference and W' to the relay. The relay recovers \hat{X}_I^n and W' successfully if $R' + I(X_I; \hat{X}_I) \leq I(X_{SR}; Y_R|X_R)$. As a result of the source-to-relay communication, the channel to the destination can be seen as a MAC with common messages in which the source and the relay have the message sets (W', W'', W_I) and (W') , respectively. The source and relay can thus employ a code in which the source codeword V^n depends on messages (W', W'', W_I) and the relay codeword U^n depends on message W' . The reason for using auxiliary codebooks instead of the actual transmitted signals X_{SD}^n and X_R^n is because unlike the corresponding conventional model, here the source and the relay also share the compressed interference information \hat{X}_I^n . In the scheme (C,S,1) at hand, this information is forwarded in an “analog” fashion to the receiver. This is accomplished by mapping the codewords V^n and U^n , obtained as discussed above, and the compressed state information \hat{X}_I^n , into the transmitted signals X_{SD}^n and X_R^n , respectively. Following the results for MAC with common messages [26] [27] [28], an achievable rate region is obtained as

$$R'' \leq I(V; Y_D|UX_I) \quad (8a)$$

$$R'' + R_I \leq I(VX_I; Y_D|U) \quad (8b)$$

$$R'' + R' \leq I(VU; Y_D|X_I) \quad (8c)$$

$$R'' + R' + R_I \leq I(VUX_I; Y_D), \quad (8d)$$

for some input pmf $P_{UVX_RX_{SD}|X_I\hat{X}_I} = P_U P_{X_R|U\hat{X}_I} P_{V|UX_I} P_{X_{SD}|V\hat{X}_I}$. Incorporating the constraint on R' above into (8) gives us (7). \square

Proposition 3.4: For Scheme (C,S,1), the following rate is achievable for the AWGN model (1):

$$R_{(C,S,1)} = \max_{\substack{r_q, \rho_{W'}, \rho_{W''}, \\ \rho_{W_I}, \bar{\rho}_{W'}, \bar{\rho}_{W''}, \bar{\rho}_{W_I}, \gamma: \\ |\rho_{W'}|, |\rho_{W''}|, |\rho_{W_I}|, \\ |\bar{\rho}_{W'}|, |\bar{\rho}_{W''}|, \gamma \in [0,1]}} \min \begin{cases} \mathcal{C}(P_{W''}) + (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - r_q)^+, \\ (\mathcal{C}(P_{W''} + P_{W_I}) - R_I)^+ + (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - r_q)^+, \\ \mathcal{C}(P_{W''} + P_{W'}), \\ (\mathcal{C}(P_{W''} + P_{W'} + P_{W_I}) - R_I)^+ \end{cases} \quad (9)$$

subject to $0 \leq r_q \leq \mathcal{C}(|h_{SR}|^2(1-\gamma)P_S)$,

$|\rho_{W'}|^2 + |\rho_{W''}|^2 + |\rho_{W_I}|^2 \leq 1$ and $|\bar{\rho}_{W'}|^2 + |\bar{\rho}_{W''}|^2 \leq 1$

where $P_{W'} = (|h_{RD}||\bar{\rho}_{W'}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2/N_{eq}$, $P_{W''} = |h_{SD}|^2|\rho_{W''}|^2\gamma P_S/N_{eq}$, $P_{W_I} =$

$(|h_{SD}||\rho_{W_I}|\sqrt{\gamma P_S} + |h_{RD}||\bar{\rho}_{W_I}|\sqrt{P_R(1-2^{-r_q})} + |h_I|\sqrt{P_I})^2/N_{eq}$ and $N_{eq} = |h_{RD}|^2|\bar{\rho}_{W_I}|^2P_R2^{-r_q} + 1$.

Sketch of the proof: The source quantizes the interference signal X_I with rate r_q using a quantization codebook with rate $I(X_I; \hat{X}_I)$. The quantization codebook is characterized by the reverse test channel $X_I = \hat{X}_I + Q$, with Q being a zero-mean complex Gaussian variable with variance $P_I2^{-r_q}$, independent of X_I , or equivalently by the test channel $\hat{X}_I = \rho X_I + Q'$, with $\rho = 1 - 2^{-r_q}$ and Q' being a complex Gaussian random variable with zero mean and variance $P_I2^{-r_q}(1 - 2^{-r_q})$, independent of X_I . The source inputs X_{SD} and X_{SR} are allocated power γP_S and $(1 - \gamma)P_S$, respectively where $0 \leq \gamma \leq 1$. We assume $X_{SD} = V$ so that the source does not forward the quantized interference \hat{X}_I . We set $X_{SD} = \rho_{W'}\sqrt{\gamma P_S}U_{W'} + \rho_{W''}\sqrt{\gamma P_S}U_{W''} + \rho_{W_I}\sqrt{\gamma P_S}U_{W_I}$, $X_R = \bar{\rho}_{W'}\sqrt{P_R}U_{W'} + k\hat{X}_I$ and $U = U_{W'}$, $k = \frac{|\bar{\rho}_{W_I}|\sqrt{P_R}}{\sqrt{\rho P_I}}$ where $U_{W'}$, $U_{W''}$, U_{W_I} are independent, zero mean, unit variance, complex Gaussian random variables and carry the messages W' , W'' and W_I , respectively. Furthermore, $U_{W'}$ and $U_{W''}$ are independent of X_I and \hat{X}_I whereas $\mathbb{E}[U_{W_I}X_I] = \sqrt{P_I}$. The destination decodes messages W' , W'' and W_I jointly. \square

Remark 3.4: Similar to Remark 3.3, when we set $r_q = 0$ in (9), Scheme (C,S,1) boils down to the special case in which the source mitigates the interference without the help of the relay using the structured approach and the relay is used for only source message cooperation.

Now, we turn to scheme (C,S,2).

Proposition 3.5: For Scheme (C,S,2), the following rate is achievable for the DM model:

$$R_{(C,S,2)} = \max \min \begin{cases} I(X_{SD}; Y_D \hat{X}_I | X_R X_I U) + (I(X_{SR}; Y_R | X_R) - I(X_I; \hat{X}_I | U Y_D))^+, \\ (I(X_{SD} X_I; Y_D \hat{X}_I | X_R U) - R_I)^+ + (I(X_{SR}; Y_R | X_R) - I(X_I; \hat{X}_I | U Y_D))^+, \\ I(X_{SD} X_R; Y_D \hat{X}_I | X_I U), \\ (I(X_{SD} X_R X_I; Y_D \hat{X}_I | U) - R_I)^+ \end{cases}, \quad (10)$$

where the maximum is over all input pmfs $P_{U \hat{X}_I X_R X_{SR} X_{SD} | X_I}$ of the form $P_{\hat{X}_I | X_I} P_{X_{SR} | X_R \hat{X}_I} P_U P_{X_R | U} P_{X_{SD} | U X_R X_I}$ such that the inequality $I(U; Y_D) \geq I(X_I; \hat{X}_I | U Y_D)$ holds.

Sketch of the proof: The source quantizes the interference signal X_I^n into a reconstruction sequence \hat{X}_I^n by using a test channel $P_{\hat{X}_I | X_I}$. Moreover, random binning is performed according to the Wyner-Ziv strategy (See, e.g., [25]), reducing the rate of the compression codebook to $I(X_I; \hat{X}_I | U Y_D)$. The source sends the index of the quantized interference and message W' to the relay. The relay recovers the compression index (but not \hat{X}_I^n) and W' successfully if

$R' + I(X_I; \hat{X}_I | UY_D) \leq I(X_{SR}; Y_R | X_R)$. The relay then maps the index of the quantized interference received from the source into a codeword U^n from an independent codebook and forwards it along with the codeword that encodes message W' to the destination. The destination first decodes the codeword U^n , which is guaranteed if $I(U; Y_D) \geq I(X_I; \hat{X}_I | UY_D)$. From the compression index, the destination can now recover \hat{X}_I^n via Wyner-Ziv decoding, since it has the side information Y_D^n and U^n . The decoded sequence \hat{X}_I^n is then used to facilitate decoding at the destination. The resulting channel to the destination is thus a MAC with common messages as (C,S,1) in which the source and the relay have the message sets (W', W'', W_I) and W' , respectively. Unlike (C,S,1), here the destination has the knowledge of both \hat{X}_I^n and U^n , which is used to jointly decode the messages set (W', W'', W_I) . Similar to (C,S,1), an achievable rate region is obtained as

$$R'' \leq I(X_{SD}; Y_D \hat{X}_I | X_R X_I U) \quad (11a)$$

$$R'' + R_I \leq I(X_{SD} X_I; Y_D \hat{X}_I | X_R U) \quad (11b)$$

$$R'' + R' \leq I(X_{SD} X_R; Y_D \hat{X}_I | X_I U) \quad (11c)$$

$$R'' + R' + R_I \leq I(X_{SD} X_R X_I; Y_D \hat{X}_I | U), \quad (11d)$$

for some input pmf $P_{UX_R X_{SD} | X_I} = P_U P_{X_R | U} P_{X_{SD} | U X_R X_I}$. Incorporating the constraints on R' and $I(U; Y_D)$ above into (11) gives us (10). \square

Proposition 3.6: For Scheme (C,S,2), the following rate is achievable for the AWGN model (1):

$$R_{(C,S,2)} = \max_{\substack{r_q, \rho_{W'}, \rho_{W''}, \\ \rho_{W_I}, \rho_U, \bar{\rho}_{W'}, \bar{\rho}_U, \gamma: \\ |\rho_{W'}|, |\rho_{W''}|, |\rho_{W_I}|, \\ |\rho_U|, |\bar{\rho}_{W'}|, |\bar{\rho}_U|, \gamma \in [0,1]}} \min \begin{cases} \mathcal{C}(P_{W''}) + (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - r_q)^+, \\ (\log_2((1+P_{W''})\frac{P_I}{D} + P_{W_I}) - R_I)^+ \\ + (\mathcal{C}(|h_{SR}|^2(1-\gamma)P_S) - r_q)^+, \\ \mathcal{C}(P_{W''} + P_{W'}), \\ (\log_2((1+P_{W''} + P_{W'})\frac{P_I}{D} + P_{W_I}) - R_I)^+ \end{cases} \quad (12)$$

$$\text{subject to } 0 \leq r_q \leq \min \begin{cases} \mathcal{C}(|h_{SR}|^2(1-\gamma)P_S), \\ \mathcal{C}\left(\frac{P_U}{P_{W'} + P_{W''} + P_{W_I} + 1}\right) \end{cases}$$

$$|\rho_{W'}|^2 + |\rho_{W''}|^2 + |\rho_{W_I}|^2 + |\rho_U|^2 \leq 1 \text{ and } |\bar{\rho}_{W'}|^2 + |\bar{\rho}_U|^2 \leq 1$$

where $P_{W'} = (|h_{RD}||\bar{\rho}_{W'}|\sqrt{P_R} + |h_{SD}||\rho_{W'}|\sqrt{\gamma P_S})^2$, $P_{W''} = |h_{SD}|^2|\rho_{W''}|^2\gamma P_S$, $P_{W_I} = (|h_{SD}||\rho_{W_I}|\sqrt{\gamma P_S} + |h_I|\sqrt{P_I})^2$, $P_U = (|h_{SD}||\rho_U|\sqrt{\gamma P_S} + |h_{RD}||\bar{\rho}_U|\sqrt{P_R})^2$, $D = P_I 2^{-r_q} \frac{(1-x)}{1-x2^{-r_q}}$ and $x = P_{W_I}/(P_{W'} + P_{W''} + P_{W_I} + 1)$.

Sketch of the proof: Similar to (C,S,1), the source quantizes the interference signal X_I with rate after binning, given by $r_q = I(X_I; \hat{X}_I | UY_D)$. The quantization codebook is characterized by the reverse test channel $X_I = \hat{X}_I + Q$, with Q being a zero-mean complex Gaussian variable with variance D , independent of X_I , or equivalently the test channel $\hat{X}_I = \rho X_I + Q'$, with $\rho = 1 - D/P_I$ and Q' being a complex Gaussian random variable with zero mean and variance $D(1 - D/P_I)$, independent of X_I . We obtain $D = P_I 2^{-r_q} \frac{(1-x)}{1-x2^{-r_q}}$ where x is defined above. The term $\frac{(1-x)}{1-x2^{-r_q}}$ represents the percentage of the decreased distortion due to side information about X_I at the destination. When $x = 0$, $D = P_I 2^{-r_q}$ which is the case where there is no side information about X_I at the destination. As $x \rightarrow 1$, $D \rightarrow 0$ for any nonzero r_q and the destination can completely recover X_I using the side information. The source inputs X_{SD} and X_{SR} are allocated power γP_S and $(1 - \gamma)P_S$, respectively where $0 \leq \gamma \leq 1$. We set $X_{SD} = \rho_{W'} \sqrt{\gamma P_S} U_{W'} + \rho_{W''} \sqrt{\gamma P_S} U_{W''} + \rho_{W_I} \sqrt{\gamma P_S} U_{W_I} + \rho_U \sqrt{\gamma P_S} U$ and $X_R = \bar{\rho}_{W'} \sqrt{P_R} U_{W'} + \bar{\rho}_U \sqrt{P_R} U$ where $U_{W'}$, $U_{W''}$, U_{W_I} and U are independent, zero mean, unit variance, complex Gaussian random variables and carry the messages W' , W'' , W_I and the index of the compressed interference, respectively. Furthermore, $U_{W'}$, $U_{W''}$ and U are independent of X_I and \hat{X}_I whereas $\mathbb{E}[U_{W_I} X_I] = \sqrt{P_I}$. The destination first decodes the codeword U and thus recovers \hat{X}_I , and then it decodes messages W' , W'' and W_I jointly using the knowledge of U and \hat{X}_I . \square

A. Discussions

For comparison purposes, we also show the performance of the Scheme Analog Input Description, referred to as AID [1] [17]. In this scheme, the source generates the codeword to be transmitted by the relay as if the relay knew the interference and the message non-causally and they used DPC jointly. The source then quantizes this codeword and sends it to the relay through the source-relay link. The relay simply forwards a scaled version of the quantized signal received from the source. The achievable rate for DM and AWGN are given in [17, Theorem 2] and [17, Theorem 4], respectively. For the DMC model, [17, Theorem 2] can be easily modified by setting $V = X_{1R}$. For Gaussian case, we incorporate complex channel gains into [1, Theorem 4] and obtain

$$R_{AID} = \max_{\gamma: \gamma \in [0,1]} \mathcal{C} \left(\frac{(|h_{SD}| \sqrt{\gamma P_S} + |h_{RD}| \sqrt{P_R - D})^2}{1 + |h_{RD}|^2 D} \right) \quad (13)$$

$$\text{where } D = \frac{P_R}{|h_{SR}|^2 (1 - \gamma) P_S + 1}.$$

A special case of the model presented in this paper is a multihop channel characterized by

$P_{Y_R Y_D | X_{SD}, X_{SR}, X_R, X_I} = P_{Y_R | X_{SR}} P_{Y_D | X_R, X_I}$. The achievable rates of this section can be easily specialized to the multihop channel. Specifically, for DM model, we remove the dependence of Y_D on X_{SD} and we set $X_{SD} = \emptyset$. For Gaussian case, we set $h_{SD} = 0$ and hence $X_{SD} = 0$. An achievable rate for the multihop channel by treating the interference as i.i.d. state was derived in [29]. This scheme, denoted by NL-DF, utilizes nested lattice codes to cancel an integer part of the interference while treating the residual of the interference as noise. The achieved rate for AWGN model can be written as [29]:

$$R_{NL-DF} = \left[\log \left(\frac{|h_{SR}|^2 |h_{RD}|^2 P_S P_R + |h_{SR}|^2 P_S + |h_{RD}|^2 P_R + 1}{|h_{SR}|^2 P_S + |h_{RD}|^2 P_R + 2} \right) \right]^+. \quad (14)$$

IV. ON THE OPTIMALITY OF INTERFERENCE FORWARDING

In this section, we consider a special case of general model considered so far where $Y_D = (Y_{D_1}, Y_{D_2})$ and the channel to the destination factorizes as

$$P_{Y_D | X_{SD}, X_R, X_I} = P_{Y_{D_1} | X_{SD}} \cdot P_{Y_{D_2} | X_R, X_I}, \quad (15)$$

as depicted in Fig. 3. This corresponds to a model where the links $S - D$ and $R - D$ are orthogonal to each other, in addition to being orthogonal to the $S - R$ channel $P_{Y_R | X_{SR}, X_R}$. In other words, this scenario can be seen as the parallel of a multihop channel $S - R - D$ and a direct channel $S - D$. Moreover, from (15), the interference affects the $R - D$ channel only. We are interested in obtaining general guidelines on how the interference information at the source should be leveraged. In particular, since the interference only affects one of the parallel channels, namely the multihop link $S - R - D$, should the $S - D$ channel be used to provide interference information so as to facilitate decoding on the $S - R - D$ link? A similar question can be of course posed for the case where interference affects only the $S - D$ link.

The question is motivated by reference [15], where it is shown that if the interference is *unstructured* and the relay is informed about the source message (but not the interference), interference information should *not* be forwarded on the $S - D$ link. A related scenario is also considered in [1], where instead *unstructured* interference affects the $S - R$ and $S - D$ links only, in a dual manner with respect to the model at hand.

We tackle the question above first for the DMC model. The next proposition shows that, even with structured interference, there is no advantage in using the $S - D$ link for interference management.

Proposition 4.1: In the model of Fig. 3, capacity is achieved by transmitting independent information on the multihop link $S - R - D$ and on the $S - D$ link. Moreover, the signal sent on the $S - D$ link can be chosen to be independent of the interference signal.

Proof: We prove this result by evaluating the capacity in multiletter form and arguing that the derived capacity can be achieved by a scheme that complies with the statement of Proposition 4.1. In particular, we prove that the capacity is given by $C = C_{SD} + C_{SRD}$, where

$$C_{SD} = \max_{P_{X_{SD}}} I(X_{SD}; Y_{D_1}) \quad (16)$$

$$\text{and } C_{SRD} = \max_{P_{X_{SR}^n|X_I^n}, P_{X_R^n|Y_{SR}^n}} I(X_{SR}^n; Y_{D_2}^n), \quad (17)$$

and $P_{X_R^n|Y_R^n} = \prod_{i=1}^n P_{X_{Ri}|Y_{SR}^{i-1}}$. Note that C_{SD} is the maximum rate achievable on the (interference-free) $S - D$ link, which is given by the standard point-to-point capacity (16), while C_{SRD} is the maximum achievable rate on the $S - R - D$ link. The latter cannot in general be calculated as a single-letter expression, unlike C_{SD} . Moreover, note that (17) is simply achieved by using the encoding strategies described by pmfs $P_{X_{SR}^n|X_I^n}$ and $P_{X_R^n|Y_R^n}$. Since by these arguments, C is achievable, we only need to prove that C is also an upper bound on the capacity. This is done in Appendix A. ■

We now specialize the result above to the corresponding Gaussian model shown in Fig. 4, which is described by the input and output relations at time instant i

$$Y_{R,i} = h_{SR,i}X_{SR,i} + Z_{R,i}, \quad Y_{D_1,i} = h_{SD,i}X_{SD,i} + Z_{D_1,i} \quad \text{and} \quad Y_{D_2,i} = h_{RD,i}X_{R,i} + h_{I,i}X_{I,i} + Z_{D_2,i} \quad (18)$$

where the noises $Z_{D_1,i}$ and $Z_{D_2,i}$ are independent zero mean complex Gaussian random variable with unit variance. The result of Proposition 4.1 can be easily generalized to a scenario with power constraints and can thus be applied also to the Gaussian model. Specifically, to simplify our results, we impose two separate power constraints on X_{SR} and X_{SD} as

$$\frac{1}{n} \sum_{i=1}^n \mathbb{E} [|X_{SR,i}|^2] \leq P_{SR} \quad \text{and} \quad \frac{1}{n} \sum_{i=1}^n \mathbb{E} [|X_{SD,i}|^2] \leq P_{SD}, \quad (19)$$

along with the relay power constraint in (2a). The following Proposition obtains the capacity for this model in a more explicit way than (16)-(17) for some special cases. Note that $C_{SD} = \mathcal{C}(|h_{SD}|^2 P_{SD})$, while C_{SRD} is generally unknown. We define

$$C'_{SRD} = \max \begin{cases} \mathcal{C}\left(\frac{|h_{RD}|^2 P_R}{1 + |h_I|^2 P_I}\right), \\ \min \{ \mathcal{C}(|h_{RD}|^2 P_R), (\mathcal{C}(|h_{RD}|^2 P_R + |h_I|^2 P_I) - R_I)^+ \} \end{cases}. \quad (20)$$

Proposition 4.2: If $\mathcal{C}(|h_{SR}|^2 P_{SR}) \geq R_I + \mathcal{C}(|h_{RD}|^2 P_R)$, then the scheme (D,U) is optimal and the capacity is given by

$$C = \mathcal{C}(|h_{SD}|^2 P_{SD}) + \mathcal{C}(|h_{RD}|^2 P_R). \quad (21)$$

If instead $\mathcal{C}(|h_{SR}|^2 P_{SR}) \leq C'_{SRD}$, then a scheme that chooses the best strategy between (N,S) and (N,U) for the given system parameters is optimal and the capacity is $C = \mathcal{C}(|h_{SD}|^2 P_{SD}) + \mathcal{C}(|h_{SR}|^2 P_{SR})$.

Proof: If $\mathcal{C}(|h_{SR}|^2 P_{SR}) \geq R_I + \mathcal{C}(|h_{RD}|^2 P_R)$, then the source can provide both interference and useful message to the relay without loss of optimality, since the rate of the message can never be larger than $\mathcal{C}(|h_{RD}|^2 P_R)$ by cut-set arguments. Scheme (D,U) is thus optimal and achieves the interference-free capacity (21). The case $\mathcal{C}(|h_{SR}|^2 P_{SR}) \leq C'_{SRD}$ is more complex. From [21] [30] it is known that the maximum rate on the R-D link, assuming that the relay is unaware of the interference is given by C'_{SRD} . This is achieved by having the destination either treat interference as noise or perform joint decoding of source information and interference. By the cut-set bound we also know that $C_{SRD} \leq \mathcal{C}(|h_{SR}|^2 P_{SR})$. However, rate $C_{SRD} = \mathcal{C}(|h_{SR}|^2 P_{SR})$ is achievable if $\mathcal{C}(|h_{SR}|^2 P_{SR}) \leq C'_{SRD}$ by not informing the destination about the interference and using the decoding strategy that attains C'_{SRD} . ■

V. ERGODIC FADING

In this section, we study the effect of ergodic fading in model (1) on the performance of the proposed schemes. We recall that the instantaneous values of the channels are only known to the receivers, while the transmitters only have knowledge of the channel statistics. As for the latter, we assume that channel gains h_{SR} , h_{SD} , h_{RD} and h_I are independent Ricean distributed with parameters K_{SR} , K_{SD} , K_{RD} , K_I , i.e., $h_{SR} = \mu_{SR} + z_{SR}$ where μ_{SR} represents the direct (deterministic) line of sight component and $z_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ such that $|\mu_{SR}|^2 + \sigma_{SR}^2 = 1$ and $|\mu_{SR}|^2 / \sigma_{SR}^2 = K_{SR}$, and likewise for other channel gains.

A. No Relay Case

We first study the point-to-point channel, i.e., where the relay is not present. This forms a foundation of the multihop relay channel investigated in Sec. V-B. For this scenario, the achievable rate with the unstructured approach is given by

$$R_U = \max_{\alpha} \mathbb{E} \left[\log_2 \left(\frac{(|h_{SD}|^2 P_S)(|h_{SD}|^2 P_S + |h_I|^2 P_I + 1)}{|h_{SD}|^2 |h_I|^2 P_S P_I (1 - 2\text{Re}(\alpha) + |\alpha|^2) + \alpha^2 |h_I|^2 P_I + |h_{SD}|^2 P_S} \right) \right]. \quad (22)$$

We employ GP coding with linear assignment of auxiliary random variable U with an inflation factor α [9]. The parameter α is chosen to be fixed for all fading levels due to the lack of CSIT and is optimized numerically, as opposed to the approaches in [7], [8] and [10].

For the structured approach, from [19], we easily obtain the achievable rate

$$R_S = \max_{\substack{\rho, \rho_I, \rho_{I'} \\ |\rho|, |\rho_I|, |\rho_{I'}| \in [0,1]}} \min \begin{cases} \mathbb{E} [\mathcal{C} (|h_{SD}\rho|^2 P_S)], \\ (\mathbb{E} [\mathcal{C} (|h_{SD}\rho|^2 P_S + |h_{SD}\rho_{I'}|^2 P_S + |h_{SD}\rho_I \sqrt{P_S} + h_I \sqrt{P_I}|^2)] - R_I)^+ \end{cases} \\ \text{subject to } |\rho|^2 + |\rho_I|^2 + |\rho_{I'}|^2 \leq 1 \quad (23)$$

where the source allocates powers for forwarding its own message and interference to the destination. In particular, power $|\rho_I|^2 P_S$ is used to transmit interference by forwarding the same codeword transmitted by the interferer, while power $|\rho_{I'}|^2 P_S$ is devoted to transmission of the interference message via an independently generated codeword. The rationale for this is that, as $K \rightarrow \infty$, fading becomes deterministic and it is optimal for the source to transmit coherently with the interferer by setting $\rho_{I'} = 0$. Instead, as $K \rightarrow 0$ (Rayleigh fading), it is more advantageous for the source to forward interference by using an independent codebook by setting $\rho_I = 0$. Hence, the source employs both of the interference forwarding strategies to accommodate intermediate K values.

B. Multihop Relay Channel

In this section, we include the relay in the ergodic fading model by considering the special case of a multihop relay channel, i.e., $h_{SD} = 0$. The detailed analysis and insights can be extended to the general orthogonal components relay channel.

The following propositions report the achievable rates of the proposed schemes for the scenario at hand. The proofs are straightforward consequences of the analysis above and Sec. III.

Proposition 5.1: For (D,U), the following rate is achievable for the multihop fading model:

$$R_{(D,U)} = \max_{\alpha} \min \begin{cases} (\mathbb{E} [\mathcal{C} (|h_{SR}|^2 P_S)] - R_I)^+, \\ \mathbb{E} \left[\log_2 \left(\frac{(|h_{RD}|^2 P_R)(|h_{RD}|^2 P_R + |h_I|^2 P_I + 1)}{|h_{RD}|^2 |h_I|^2 P_R P_I (1 - 2Re(\alpha) + |\alpha|^2) + \alpha^2 |h_I|^2 P_I + |h_{RD}|^2 P_R} \right) \right] \end{cases} \quad (24)$$

Proposition 5.2: For (D,S), the following rate is achievable for the multihop fading model:

$$R_{(D,S)} = \max_{\substack{\bar{\rho}, \bar{\rho}_I, \bar{\rho}_{I'} \\ |\bar{\rho}|, |\bar{\rho}_I|, |\bar{\rho}_{I'}| \in [0,1]}} \min \begin{cases} (\mathbb{E} [\mathcal{C} (|h_{SR}|^2 P_S)] - R_I)^+, \\ \mathbb{E} [\mathcal{C} (|h_{RD}\bar{\rho}|^2 P_R)], \\ (\mathbb{E} [\mathcal{C} (|h_{RD}\bar{\rho}|^2 P_R + |h_{RD}\bar{\rho}_{I'}|^2 P_R + |h_{RD}\bar{\rho}_I \sqrt{P_R} + h_I \sqrt{P_I}|^2)] - R_I)^+ \end{cases} \\ \text{subject to } |\bar{\rho}|^2 + |\bar{\rho}_I|^2 + |\bar{\rho}_{I'}|^2 \leq 1 \quad (25)$$

Remark 5.1: For Gaussian model (1), structured strategies in no-relay case as well as in (D,S) are inferior to the unstructured ones in no-fading case due to the ability of DPC completely eliminating the effect of the interference. However as shown in Section VI, these strategies become meaningful under fading where precoding can not completely cancel the interference.

Proposition 5.3: For (C,U), the following rate is achievable for the multihop fading model:

$$R_{(C,U)} = \max_{r_q, \alpha} \min \left\{ \begin{aligned} & (\mathbb{E} [\mathcal{C}(|h_{SR}|^2 P_S)] - r_q)^+, \\ & \mathbb{E} \left[\log_2 \left(\frac{(|h_{RD}|^2 P_R)(|h_{RD}|^2 P_R + |h_I|^2 (P_I - D) + N_{eq})}{|h_{RD}|^2 |h_I|^2 P_R (P_I - D) (1 - 2\text{Re}(\alpha) + |\alpha|^2) + N_{eq}(\alpha^2 |h_I|^2 (P_I - D) + |h_{RD}|^2 P_R)} \right) \right] \end{aligned} \right\} \quad (26)$$

where $N_{eq} = |h_I|^2 D + 1$ and $D = P_I 2^{-r_q}$.

Proposition 5.4: For (C,S,1), the following rate is achievable for the multihop fading model:

$$R_{(C,S,1)} = \max_{\substack{r_q, \bar{\rho}, \bar{\rho}_I, \bar{\rho}_{I'}: \\ |\bar{\rho}|, |\bar{\rho}_I|, |\bar{\rho}_{I'}| \in [0,1]}} \min \left\{ \begin{aligned} & (\mathbb{E} [\mathcal{C}(|h_{SR}|^2 P_S)] - r_q)^+, \\ & \mathbb{E} [\mathcal{C}(|h_{RD} \bar{\rho}|^2 P_R / N_{eq})], \\ & (\mathbb{E} [\mathcal{C}((|h_{RD} \bar{\rho}|^2 P_R + |h_{RD} \bar{\rho}_{I'}|^2 P_R (1 - 2^{-r_q}) + \\ & |h_{RD} \bar{\rho}_I \sqrt{P_R (1 - 2^{-r_q})} + h_I \sqrt{P_I}|^2) / N_{eq})] - R_I)^+ \end{aligned} \right. \quad (27)$$

subject to $|\bar{\rho}|^2 + |\bar{\rho}_I|^2 + |\bar{\rho}_{I'}|^2 \leq 1$

where $N_{eq} = |h_{RD} \bar{\rho}_I|^2 P_R 2^{-r_q} + |h_{RD} \bar{\rho}_{I'}|^2 P_R 2^{-r_q} + 1$.

Proposition 5.5: For (C,S,2), the following rate is achievable for the multihop fading model:

$$R_{(C,S,2)} = \max_{\substack{r_q, \bar{\rho}, \bar{\rho}_U: \\ |\bar{\rho}|, |\bar{\rho}_U| \in [0,1]}} \min \left\{ \begin{aligned} & (\mathbb{E} [\mathcal{C}(|h_{SR}|^2 P_S)] - r_q)^+, \\ & \mathbb{E} [\mathcal{C}(|h_{RD} \bar{\rho}|^2 P_R)], \\ & (\mathbb{E} [\log_2 ((|h_{RD} \bar{\rho}|^2 P_R + 1) 2^{r_q} + |h_I|^2 P_I)] - R_I)^+ \end{aligned} \right. \quad (28)$$

subject to $|\bar{\rho}|^2 + |\bar{\rho}_U|^2 \leq 1$ and $r_q \leq \mathbb{E} \left[\mathcal{C} \left(\frac{|h_{RD} \bar{\rho}_U|^2 P_R}{|h_{RD} \bar{\rho}|^2 P_R + |h_I|^2 P_I + 1} \right) \right]$

Remark 5.2: In the fading scenario for Scheme (C,S,2), the source does not know h_{RD} and thus can not determine the instantaneous Wyner-Ziv compression rate to compress X_I with respect to the destination observation. Therefore, for simplicity, we assume that the source neglects the side information available at the destination and does not perform binning. Recall that neglecting the side information corresponds to the case where $x = 0$ in (12).

Proposition 5.6: For AID, the following rate is achievable for the multihop fading model:

$$R_{AID} = \max_{\alpha} \mathbb{E} \left[\log_2 \left(\frac{(|h_{RD}|^2 (P_R - D))(|h_{RD}|^2 (P_R - D) + |h_I|^2 P_I + N_{eq})}{|h_{RD}|^2 |h_I|^2 (P_R - D) P_I (1 - 2\text{Re}(\alpha) + |\alpha|^2) + N_{eq}(\alpha^2 |h_I|^2 P_I + |h_{RD}|^2 (P_R - D))} \right) \right] \quad (29)$$

where $N_{eq} = |h_{RD}|^2 D + 1$, $D = P_R 2^{-r_q}$ and $r_q = \mathbb{E} [\mathcal{C}(|h_{SR}|^2 P_S)]$. The source evaluates the signal to be transmitted by the relay when the relay utilizes the unstructured approach, namely DPC for

$(R - D)$ ergodic channel. The source quantizes the corresponding signal with rate r_q and forwards it to the relay. The relay simply forwards the received signal to destination.

VI. NUMERICAL RESULTS

In this section, we numerically evaluate the achievable rates for the AWGN models, both with no fading and with ergodic fading, and compare them with two following simple schemes.

- *Scheme No Relay (NR)*: The achieved rate is given by [4] and denoted as R_{NR} ;
- *Scheme No Interference (NI)*: We set $P_I = 0$ and $R_I = 0$, so that the interference is not present.

The capacity for this scenario, R_{NI} , is achieved by PDF [23] and is given by (5) with $R_I = 0$.

Note that R_{NI} provides an upper bound to rates of the proposed achievable schemes.

A. No Fading

We first consider the no fading case. In Fig. 5, the achievable rates are illustrated as a function of the interference power P_I for $P_S = P_R = 10dB$, $|h_{SD}| = |h_{SR}| = |h_{RD}| = |h_I| = 1$ and $R_I = 1$ bits/channel use. Scheme (C,U) outperforms all others for low interference power, since in this case cooperative interference mitigation strategies are not worth the capacity needed on the source-relay link for digital interference sharing. Moreover, leveraging the interference structure is not useful since interference decoding at the destination is hindered by the low interference power. For large P_I , Scheme (C,S,2) instead outperforms all others and eventually meets the upper bound R_{NI} . The larger P_I is, the less power the source and the relay need to make the interference decodable at the destination. In fact if P_I is sufficiently large, the destination is able to decode the interference without the help of the source or the relay and the schemes which utilize structured approach, namely (C,S,1) and (C,S,2) achieve interference-free bound and hence they are optimal. We also note that as the interference power increases, Schemes (C,S,1) and (C,S,2) perform the same and have $r_q = 0$ which means that the relay is utilized only for forwarding the source message. Scheme (D,U) completely eliminates the interference by MU-DPC when R_I is greater than the capacity of the source-relay link, as is the case here, and hence, the performance of Scheme (D,U) is independent of the interference power. However, there is a gap between the performance of Scheme NI and Scheme (D,U) due to the source-relay capacity used for informing the relay about the interference. Similarly the performance of the scheme (AID) also does not depend on the interference power.

In Fig. 6, we set the source-relay channel gain to $|h_{SR}| = 2$ and keep the rest of the parameters same as Fig. 5 in order to study the effects of a higher gain for source-relay channel. We observe

that Scheme (D,U) outperforms all schemes for moderate interference power P_I . Now the source and the relay are able to better mitigate the interference jointly since the (S-R) channel has enough capacity for conveying digital interference information to the relay. In fact for large $|h_{SR}|$, the capacity of source-relay channel is high enough to share the interference with the relay digitally at no extra cost and Scheme (D,U) achieves the interference-free upper bound. In Fig. 7, we increase the interference rate and set $R_I = 3$ bits/channel use by keeping the rest of the parameters the same as for Fig. 6. We observe that (AID) outperforms (D,U) as well as all other schemes for moderate interference power. Since the interference rate is large compared to the source relay channel capacity, informing the relay about the interference in a digital fashion becomes too costly.

Finally, we illustrate the achievable rate as a function of R_I in Fig. 8 for the multihop relay channel $|h_{SD}| = 0$ and we set $P_S = P_R = P_I = 10dB$ and $|h_{SR}| = |h_{RD}| = |h_I| = 1$. We also include Scheme NL-DF whose performance is independent of R_I . For small interference rate, schemes that exploit the interference structure at the destination, namely (C,S,1) and (C,S,2), result in the best rate and achieve no-interference upper bound. As the interference rate increases, schemes (D,U), (C,S,1) and (C,S,2) degrade in performance since it is harder to decode the interference at either the relay or the destination. Note also that for moderate interference rates, Scheme (C,S,2) outperforms all others, showing that interference sharing via compressed information along with a structured approach is the most beneficial strategy in this regime.

B. Ergodic Fading

In this section, we turn to fading channels. We first consider the point-to-point case, i.e., $h_{SR} = h_{RD} = 0$. In Fig. 9, we illustrate the rate as a function of the interference power for $P_S = 5dB$ and Ricean factor $K = 1$ for both h_{SD} and h_I channel gains. As the interference power increases, the structured approach outperforms the unstructured one. Recall that, in the case of no fading unstructured approach, namely DPC, achieves the no-interference upper bound and hence is optimal. However, for fading channels with no channel knowledge at the source, the unstructured approach is not able to completely cancel the interference anymore, and the structured approach becomes beneficial when the interference power is large. To get further insights on this, in Fig. 10, the rate as a function of parameter K , common for h_{SD} and h_I , is illustrated for various interference rates when $P_S = P_I = 5dB$. We observe that as K increases, the gap between the no-interference

upper bound and the performance of the unstructured approach decreases and, as $K \rightarrow \infty$, the unstructured approach achieves the no-interference bound. This is expected since, as $K \rightarrow \infty$, the channel model becomes equivalent to the no-fading case. For small K , instead, the structured approach outperforms the structured approach for small R_I .

Finally, we study multihop relay channel where $h_{SD} = 0$ and h_{SR} , h_{RD} and h_I are Ricean distributed with the same parameter K . In Fig. 11, the rate as a function of interference power is illustrated when $P_S = 10\text{dB}$, $P_R = 7\text{dB}$, $R_I = 0.4$ bits/channel use and $K = 1$. We do not include Scheme (C,S,1) in Fig. 11 since it is dominated by Scheme (C,S,2) for the chosen parameters. Since the source has more power than the relay, the second hop is the bottleneck. Therefore, interference management in the second hop becomes critical and the relay should be informed about the interference. Also, for this scenario digital interference sharing performs better than compressed interference sharing. Comparing Schemes (D,U) and (D,S), we observe that while in the no fading case (D,S) is always inferior to (D,U), under fading this is no longer true and Scheme (D,S) outperforms (D,U) for large interference power.

VII. CONCLUSION

A relay channel with orthogonal components that is corrupted by a single external interferer is studied. The interference is non-causally available only at the source, but not at the relay or at the destination. The interference is assumed to be structured, since it corresponds to a codeword of the codebook of the interferer, whose transmission strategy is assumed to be fixed. We complement previous work that studied the model under the assumption of unstructured interference by establishing achievable schemes that leverage the interference structure. Effective interference management calls, on the one hand, for appropriate communication strategies towards the relay in order to enable cooperative interference management, and, on the other, for the design of joint encoding/decoding strategies. Our work sheds light on the optimal design for DMC and AWGN channels with and without fading. The best available transmission strategies turn out to depend critically on the parameters of the interference signal (such as interference power and transmission rate) and on the channel model.

APPENDIX A

PROOF OF PROPOSITION 4.1

From Fano's inequality, we have $H(W|Y_{D1}^n, Y_{D2}^n) \leq n\epsilon_n$, where $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$, if the probability of error goes to zero as $n \rightarrow \infty$, and thus

$$nR \leq I(W; Y_{D1}^n Y_{D2}^n) + n\epsilon_n \quad (30a)$$

$$= I(W; Y_{D1}^n | Y_{D2}^n) + I(W; Y_{D2}^n) + n\epsilon_n \quad (30b)$$

$$= h(Y_{D1}^n | Y_{D2}^n) - h(Y_{D1}^n | Y_{D2}^n, W) + h(Y_{D2}^n) - h(Y_{D2}^n | W) + n\epsilon_n \quad (30c)$$

$$\leq h(Y_{D1}^n) - h(Y_{D1}^n | X_{SD}^n, Y_{D2}^n, W) + h(Y_{D2}^n) - h(Y_{D2}^n | X_{SR}^n, W) + n\epsilon_n \quad (30d)$$

$$= h(Y_{D1}^n) - h(Y_{D1}^n | X_{SD}^n) + h(Y_{D2}^n) - h(Y_{D2}^n | X_{SR}^n) + n\epsilon_n \quad (30e)$$

$$\leq nI(X_{SD}; Y_{D1}) + nI(X_{SR}^n; Y_{D2}^n) + n\epsilon_n \quad (30f)$$

where we have used the chain rule of mutual information in (30b), the fact that conditioning reduces entropy [24] in (30d) and the Markov chains $(Y_{D2}^n, W) - X_{SD}^n - Y_{D1}^n$ and $W - X_{SR}^n - Y_{D2}^n$ in (30e). In (30f), we used the same steps in the standard converse of a point-to-point channel which shows that $h(Y_{D1}^n) - h(Y_{D1}^n | X_{SD}^n) \leq nI(X_{SD}; Y_{D1})$ for $X_{SD} = X_{SD,Q}$ and $Y_{D1} = Y_{D1,Q}$ with Q being a uniformly distributed random variable in the set $[1, \dots, n]$ [24]. This concludes the proof.

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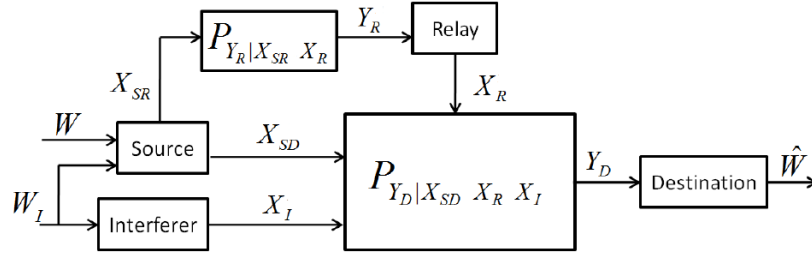


Fig. 1. Relay channel with orthogonal components under structured interference known at the source.

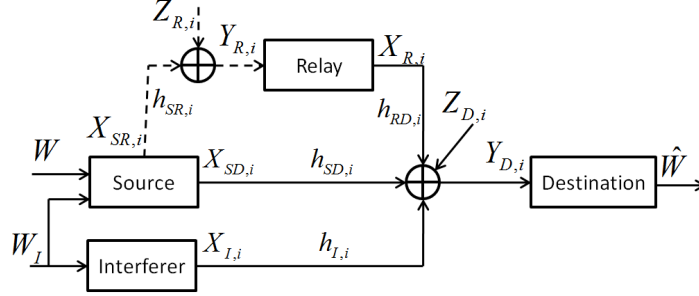


Fig. 2. AWGN relay channel with orthogonal components under structured interference known at the source where the dashed line denotes the out-of-band channel between the source and the relay.

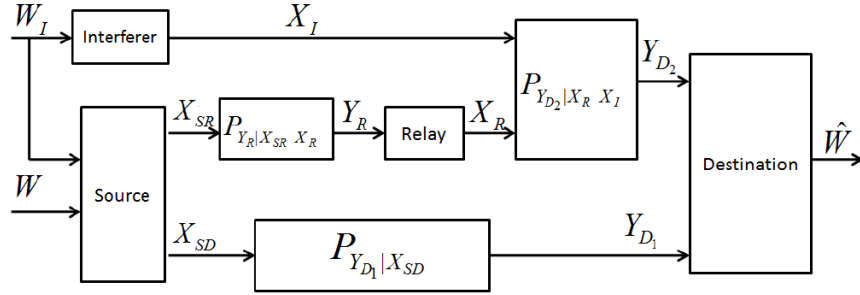


Fig. 3. Special class of relay channel with orthogonal components under structured interference known at the source.

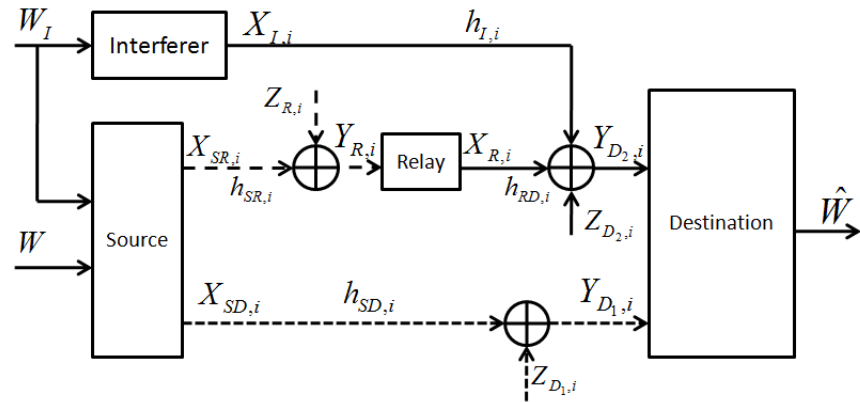


Fig. 4. Special class of AWGN relay channel with orthogonal components under structured interference known at the source for independent sources.

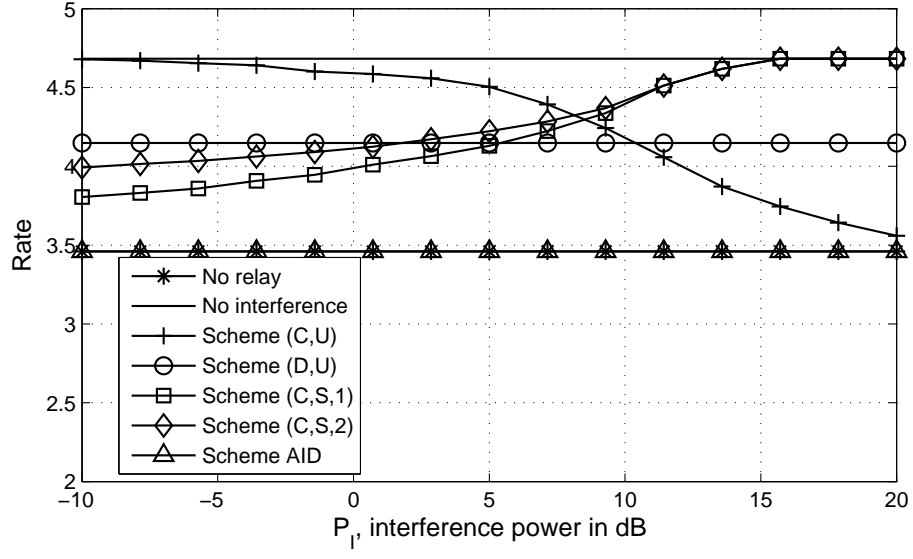


Fig. 5. Achievable rate as a function of P_I when $P_S = P_R = 10\text{dB}$, $|h_{SD}| = |h_{SR}| = |h_{RD}| = |h_I| = 1$ and $R_I = 1$.

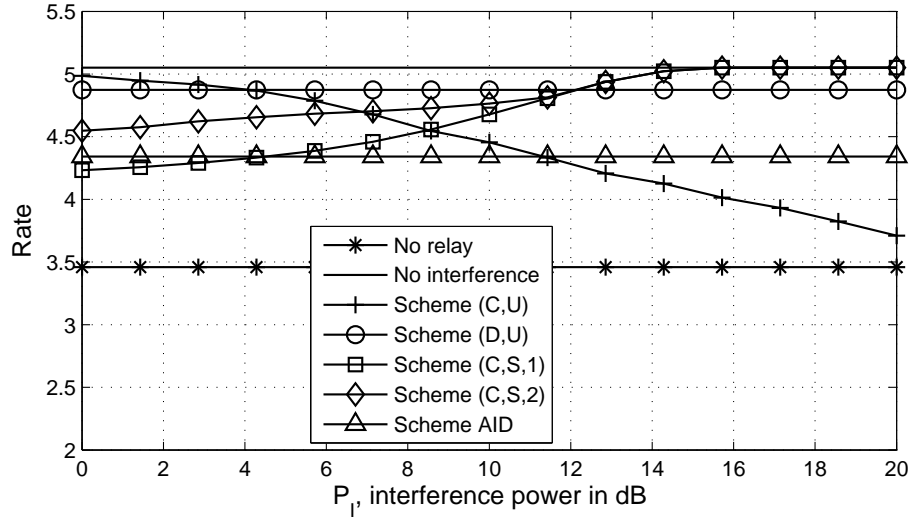


Fig. 6. Achievable rate as a function of P_I when $P_S = P_R = 10\text{dB}$, $|h_{SR}| = 2$, $|h_{SD}| = |h_{RD}| = |h_I| = 1$ and $R_I = 1$.

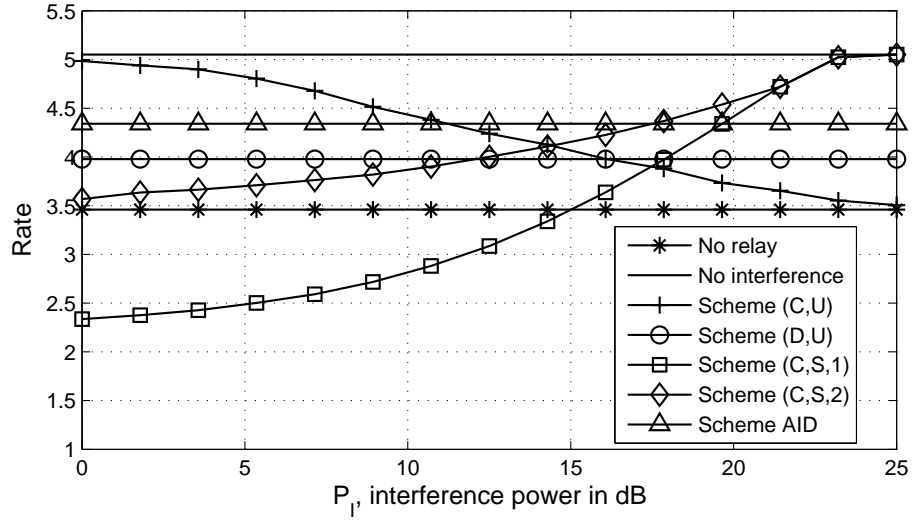


Fig. 7. Achievable rate as a function of P_I when $P_S = P_R = 10\text{dB}$, $|h_{SR}| = 2$, $|h_{SD}| = |h_{RD}| = |h_I| = 1$ and $R_I = 3$.

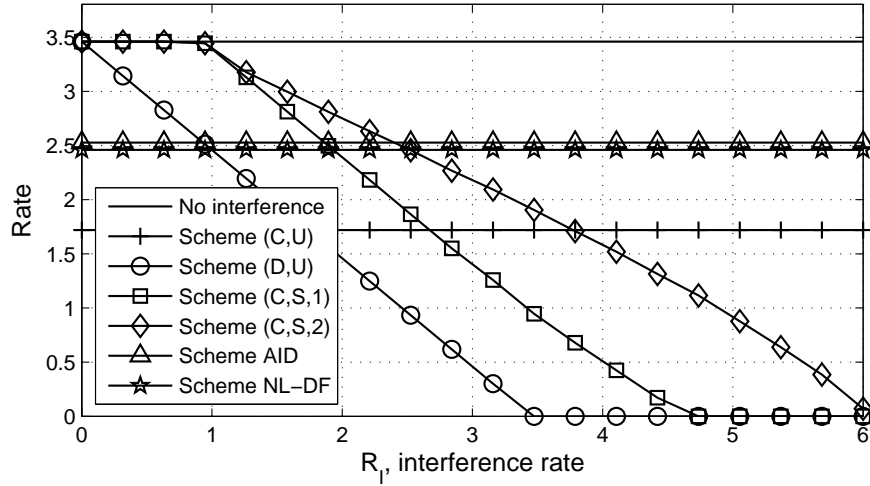


Fig. 8. Achievable rate as a function of R_I for the multihop channel ($h_{SD} = 0$) when $P_S = P_R = P_I = 10\text{dB}$, $|h_{SR}| = |h_{RD}| = |h_I| = 1$.

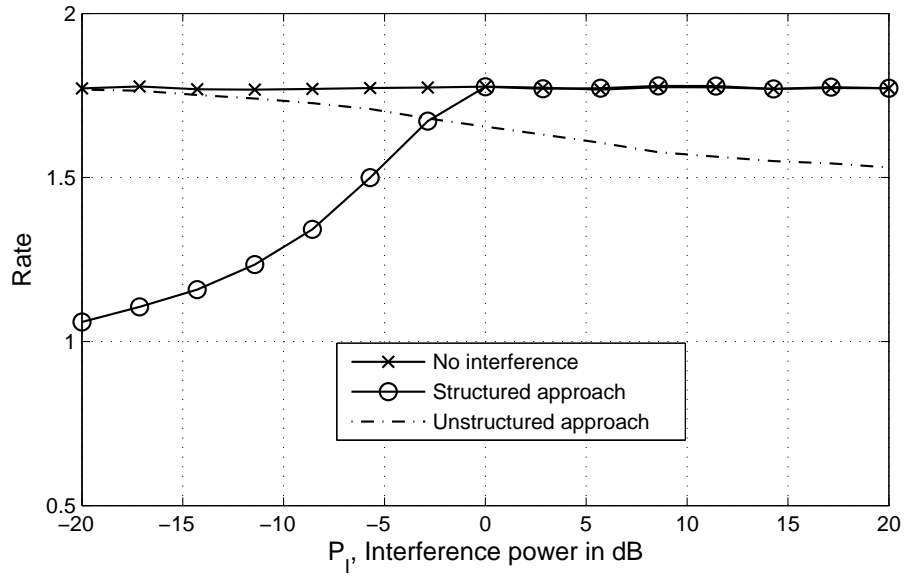


Fig. 9. Achievable rate as a function of P_I for point to point fading channel with no CSIT when $P_S = 5dB$, $K = 1$.

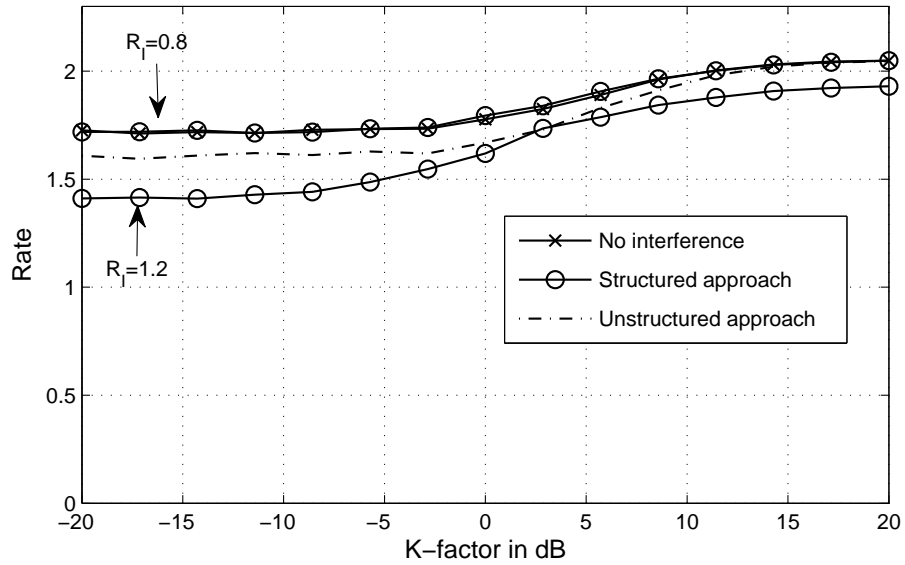


Fig. 10. Achievable rate as a function of K-factor for point to point fading channel with no CSIT for various interference rates when $P_S = P_I = 5dB$.

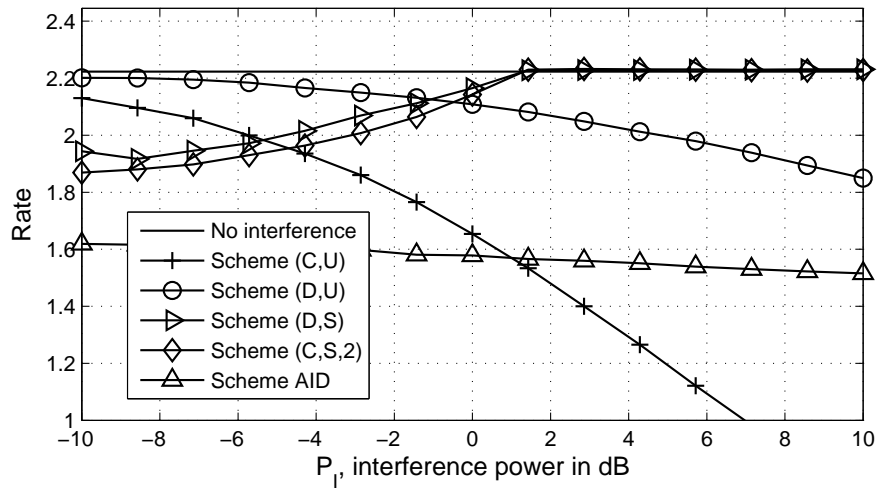


Fig. 11. Achievable rate as a function of P_I for multihop fading channel with no CSIT when $P_S = 10\text{dB}$, $P_R = 7\text{dB}$, $R_I = 0.4$, $K = 1$ and $N = 1$.